2025 MAA AMC 12B

Problem 1

The instructions on a 350-gram bag of coffee beans say that proper brewing of a large mug of pour-over coffee requires 20 grams of coffee beans. What is the greatest number of properly brewed large mugs of coffee that can be made from the coffee beans in that bag?

一袋重量为 350 克的咖啡豆的使用说明上写着，冲泡一大杯手冲咖啡需要 20 克咖啡豆．问依照标准，用这袋咖啡豆最多可以冲泡多少杯大杯咖啡?

(A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 20

Problem 2

Jerry wrote down the ones digit of each of the first 2025 positive squares: 1,4,9,6,5,6, ... What is the sum of all the numbers Jerry wrote down?

Jerry 写下了前 2025 个正整数平方的个位数字：1，4，9，6，5，6，….问 Jerry 写下的所有数的总和是多少?

(A) 9025 **(B)** 9070 **(C)** 9090 **(D)** 9115 **(E)** 9160

Problem 3

What is the value of $i(i-1)(i-2)(i-3)$, where $i=\sqrt{-1}$?

已知 $i=\sqrt{-1}$，问 $i(i-1)(i-2)(i-3)$ 的值是多少?

(A) $6-5i$ **(B)** $-10i$ **(C)** $10i$ **(D)** -10 **(E)** 10

Problem 4

The value of the two-digit number $\underline{a}\,\underline{b}$ in base seven equals the value of the two-digit number $\underline{b}\,\underline{a}$ in base nine. What is $a+b$?

七进制的两位数 $\underline{a}\,\underline{b}$ 的值等于九进制的两位数 $\underline{b}\,\underline{a}$ 的值．问 $a+b$ 是多少?

(A) 7 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 14

Problem 5

Positive integers x and y satisfy the equation $57x + 22y = 400$. What is the least possible value of $x + y$?

正整数 x 与 y 满足方程 $57x + 22y = 400$，问 $x + y$ 的最小可能值是多少?

(A) 10 **(B)** 11 **(C)** 13 **(D)** 14 **(E)** 15

Problem 6

Emmy says to Max, "I ordered 36 math club sweatshirts today." Max asks, "How much did each shirt cost?" Emmy responds, "I'll give you a hint. The total cost was $\$\underline{A}\,\underline{B}\,\underline{B}.\,\underline{B}\,\underline{A}$, where A and B are digits and $A \neq 0$." After a pause, Max says, "That was a good price." What is $A + B$?

Emmy 对 Max 说:“我今天订了 36 件数学社的运动衫.” Max 问:“每件运动衫多少钱?” Emmy 回答说:“我给你一个提示.总成本是 $\$\underline{A}\,\underline{B}\,\underline{B}.\,\underline{B}\,\underline{A}$，其中 A 和 B 是数字且 $A \neq 0$.” 停顿了一下后，Max 说:“这个价格很不错.”问 $A + B$ 的值是?

(A) 7 **(B)** 8 **(C)** 11 **(D)** 14 **(E)** 15

Problem 7

What is the value of

$$\sum_{n=2}^{255} \frac{\log_2\left(1+\frac{1}{n}\right)}{\left(\log_2 n\right)\left(\log_2(n+1)\right)}$$

问和式

$$\sum_{n=2}^{255} \frac{\log_2\left(1+\frac{1}{n}\right)}{\left(\log_2 n\right)\left(\log_2(n+1)\right)}$$

的值是多少?

(A) $\frac{3}{4}$ **(B)** $1-\frac{1}{\log_2 255}$ **(C)** $\frac{7}{8}$ **(D)** $\frac{15}{16}$ **(E)** 1

Problem 8

There are integers a and b such that the polynomial x^3-5x^2+ax+b has $4+\sqrt{5}$ as a root. What is $a+b$?

存在整数 a 与 b，使得 $4+\sqrt{5}$ 是多项式 x^3-5x^2+ax+b 的根，问 $a+b$ 是多少?

(A) 13 **(B)** 17 **(C)** 20 **(D)** 30 **(E)** 68

Problem 9

What is the tens digit of 6^{6^6}?

问 6^{6^6}的十位数字是多少?

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

Problem 10

The altitude to the hypotenuse of a $30-60-90^\circ$ right triangle is divided into two segments of lengths $x<y$ by the median to the shortest side of the triangle. What is the ratio $\frac{x}{x+y}$?

在一个 $30-60-90^\circ$ 的直角三角形中，斜边上的高被该三角形较短直角边上的中线分成两条线段，长度分别为 $x<y$. 问比值 $\frac{x}{x+y}$ 是多少?

(A) $\frac{3}{7}$ **(B)** $\frac{\sqrt{3}}{4}$ **(C)** $\frac{4}{9}$ **(D)** $\frac{5}{11}$ **(E)** $\frac{4\sqrt{3}}{15}$

Problem 11

Nine athletes, no two of whom are the same height, try out for the basketball team. One at a time, they draw a wristband at random, without replacement, from a bag containing 3 blue bands, 3 red bands, and 3 green bands. They are divided into a blue group, a red group, and a green group. The tallest member

of each group is named the group captain. What is the probability that the group captains are the three tallest athletes?

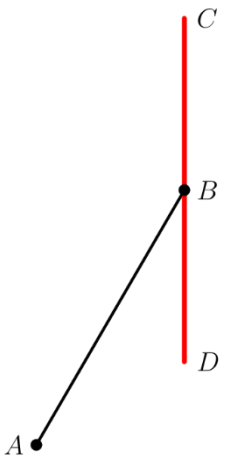
九名身高各不相同的运动员参加篮球队选拔．他们依次从装有 3 条蓝色腕带，3 条红色腕带，3 条绿色腕带的袋子中随机抽取一条腕带，且抽取后不放回．随后他们按腕带颜色被分成蓝队，红队，绿队．每队中身高最高的成员被任命为队长．问三名队长是三名最高的运动员的概率是多少?

(A) $\frac{2}{9}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{9}{28}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{3}{8}$

Problem 12

The windshield wiper on the driver's side of a large bus is depicted below.Arm $\overline{AB}$ pivots back and forth around point A, sweeping out an arc of 60°, symmetric about the vertical line through A. The wiper blade $\overline{CD}$ is attached to B at its midpoint and stays vertical as the arm moves. The arm is 3 feet long, and the wiper blade is 3.5 feet tall. What is the area of the windshield cleaned by the wiper, in square feet, to the nearest hundredth? (Assume that the windshield is a flat vertical surface.)

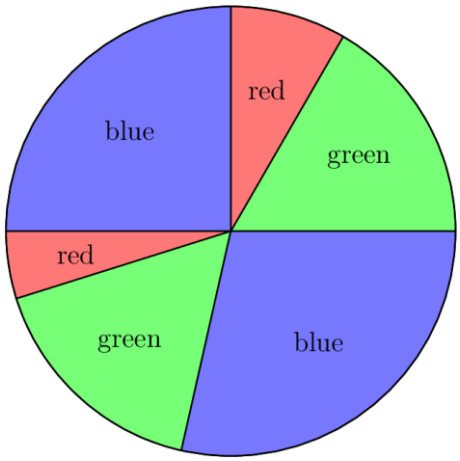
大型巴士驾驶员一侧的挡风玻璃雨刷器如下图所示．摆臂 AB 以点 A 为轴心来回摆动，扫过一个 60° 的圆弧，并且该圆弧关于通过 A 的竖直线对称．雨刷片 CD 在其中点 B 处与摆臂相连，并在摆臂转动过程中始终保持垂直．已知摆臂长 3 英尺，雨刷片高 3.5 英尺．问雨刷所能清洁的挡风玻璃的面积是多少平方英尺?（答案精确到百分位，假设挡风玻璃是平坦的垂直表面．）



(A) 9.68 **(B)** 10.14 **(C)** 10.50 **(D)** 11.32 **(E)** 12.00

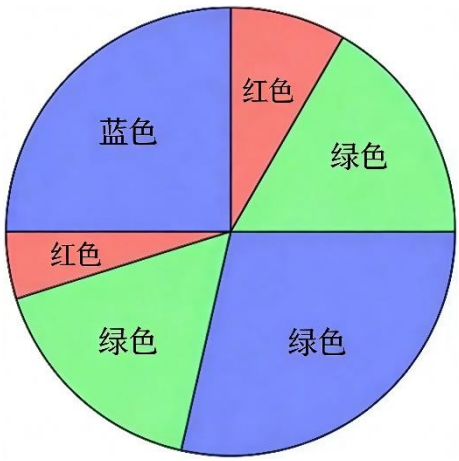
Problem 13

A circle has been divided into 6 sectors of different sizes. Then 2 of the sectors are painted red, 2 painted green, and 2 painted blue so that no two neighboring sectors are painted the same color. One such coloring is shown below.



How many different colorings are possible?

一个圆被分成了 6 个大小不同的扇形．然后将其中 2 个扇形涂成红色，2 个涂成绿色，2 个涂成蓝色，使得任意两个相邻的扇形颜色都不同．下图展示了一种这样的着色方式．



问共有多少种不同的着色方式?

(A) 12 **(B)** 16 **(C)** 18 **(D)** 24 **(E)** 28

Problem 14

Consider a decreasing sequence of n positive integers $x_1 > x_2 > \cdots > x_n$ that satisfies the following conditions:

- The average of the first 3 terms in the sequence is 2025.
- For all $4 \leq k \leq n$, the average of the first k terms is 1 less than the average of the first $k-1$ terms.

What is the greatest possible value of n?

考虑由 n 个正整数组成的递减数列 $x_1 > x_2 > \cdots > x_n$，它满足以下条件：

- 数列前 3 项的平均值是 2025.
- 对于所有 $4 \leq k \leq n$，前 k 项的平均值比前 $k-1$ 项的平均值小 1.

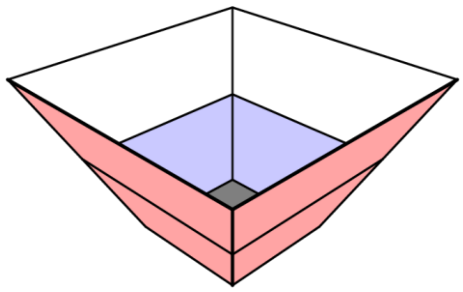
问 n 的最大可能值是多少？

(A) 1013 **(B)** 1014 **(C)** 1016 **(D)** 2016 **(E)** 2025

Problem 15

A container has a 1×1 square bottom, a 3×3 open square top, and four congruent trapezoidal sides, as shown. Starting when the container is empty, a hose that runs water at a constant rate takes 35 minutes to fill the container up to the midline of the trapezoids. How many more minutes will it take to fill the remainder of the container?

如图所示，容器的底部是 1×1 的正方形，顶部是 3×3 的开放正方形，还有四个全等的梯形侧面. 开始时容器是空的，用一根水管以恒定速率向容器中注水，需要 35 分钟才能将水注满至梯形侧面中线的高度. 问还需要多少分钟才能将容器的剩余部分注满？



(A) 79 **(B)** 85 **(C)** 90 **(D)** 95 **(E)** 105

Problem 16

An analog clock starts at midnight and runs for 2025 minutes before stopping.

What is the tangent of the acute angle between the hour hand and the minute hand when the clock stops?

指针式时钟从午夜开始运行，在经过 2025 分钟后停止．问当时钟停止时，时针与分针所成锐角的正切值是多少?

(A) 0 **(B)** $\sqrt{2}-1$ **(C)** $2-\sqrt{2}$ **(D)** $\frac{\sqrt{2}}{2}$ **(E)** $3-\sqrt{2}$

Problem 17

Each of the 9 squares in a 3×3 grid is to be colored red, blue, or yellow in such a way that each red square shares an edge with at least one blue square, each blue square shares an edge with at least one yellow square, and each yellow square shares an edge with at least one red square. Colorings that can be obtained from one another by rotations and/or reflections are to be considered the same. How many different colorings are possible?

对于 3×3 方格表中的 9 个方格，每个方格都要涂上红色、蓝色或黄色，且需满足以下条件：每个红色方格都与至少一个蓝色方格有公共边，每个蓝色方格都与至少一个黄色方格有公共边，每个黄色方格都与至少一个红色方格有公共边．通过旋转、反射或者它们的组合可以互相得到的涂色方式视为相同的，问共有多少种不同的涂色方式?

(A) 3 **(B)** 9 **(C)** 12 **(D)** 18 **(E)** 27

Problem 18

Awnik repeatedly plays a game that has a probability of winning of $\frac{1}{3}$. The outcomes of the games are independent. What is the expected value of the number of games he will play until he has both won and lost at least once?

Awnik 反复进行一项每局获胜概率为 $\frac{1}{3}$ 的游戏，各局游戏的结果互相独立．直到他至少获胜一局且至少输过一局需要进行的游戏局数的期望值是多少?

(A) $\frac{5}{2}$ **(B)** 3 **(C)** $\frac{16}{5}$ **(D)** $\frac{7}{2}$ **(E)** $\frac{15}{4}$

Problem 19

A rectangular grid of squares has 141 rows and 91 columns. Each square has room for two numbers. Horace and Vera each fill in the grid by putting the numbers from 1 through $141 \cdot 91 = 12{,}831$ into the squares. Horace fills the grid horizontally: he puts 1 through 91 in order from left to right into row 1, puts 92 through 182 into row 2 in order from left to right, and continues similarly through row 141. Vera fills the grid vertically: she puts 1 through 141 in order from top to bottom into column 1, then 142 through 282 into column 2 in order from top to bottom, and continues similarly through column 91. How many squares get two copies of the same number?

考虑有 141 行与 91 列的矩形方格表，每个方格中可填入两个数．Horace 与 Vera 分别将从 1 到 $141 \cdot 91 = 12{,}831$ 的所有数填入方格中．Horace 沿水平方向填写：他在第 1 行从左到右依序填入 1 至 91，在第 2 行从左到右依序填入 92 至 182，并以此类推直至填完第 141 行．Vera 沿垂直方向填写：她在第 1 列从上到下依序填入 1 至 141，在第 2 列从上到下依序填入 142 至 282，并以此类推直至填完第 91 列．问有多少个方格中两人填入的数相同？

(A) 7 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 19

Problem 20

A frog hops along the number line according to the following rules.

- It starts at 0.
- If it is at 0, then it moves to 1 with probability $\frac{1}{2}$ and it disappears with probability $\frac{1}{2}$.
- For $n = 1$, 2 or 3, if it is at n, then it moves to $n + 1$ with probability $\frac{1}{4}$, it moves to $n - 1$ with probability $\frac{1}{4}$, and it disappears with probability $\frac{1}{2}$.

What is the probability that the frog reaches 4?

一只青蛙按照如下规则在数轴上跳跃：

- 它从 0 出发。
- 若它在 0 处，以 $\frac{1}{2}$ 的概率跳到 1，以 $\frac{1}{2}$ 的概率消失。
- 当$n = 1$，2，3，若它在 n 处，则以 $\frac{1}{4}$ 的概率跳到 $n + 1$；以 $\frac{1}{4}$ 的概率跳到 $n - 1$；以 $\frac{1}{2}$ 的概率跳到消失；

问青蛙最终到达 4 的概率是多少?

(A) $\frac{1}{101}$ **(B)** $\frac{1}{100}$ **(C)** $\frac{1}{99}$ **(D)** $\frac{1}{98}$ **(E)** $\frac{1}{97}$

Problem 21

Two non-congruent triangles have the same area. Each triangle has sides of length 8 and 9, and the third side of each triangle has integer length. What is the sum of the lengths of the third sides?

两个不全等的三角形面积相同. 每个三角形都有两条边, 长度分别为 8 和 9, 且每个三角形的第三条边的长度均为整数. 问这两个第三条边的长度之和是多少?

(A) 20 **(B)** 22 **(C)** 24 **(D)** 26 **(E)** 28

Problem 22

What is the greatest possible area of the triangle in the complex plane with vertices $2z$, $(1+i)z$, and $(1-i)z$, where z is a complex number satisfying $|4z-2|=1$?

在复数平面中, 有一个三角形的顶点分别为 $2z$, $(1+i)z$, $(1-i)z$, 其中 z 是满足 $|4z-2|=1$ 的复数. 问此三角形的面积最大可能是多少?

(A) $\frac{1}{4}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{9}{16}$ **(D)** $\frac{3}{4}$ **(E)** 1

Problem 23

Let S be the set of all integers $z>1$ such that for all pairs of nonnegative integers (x,y) with $x<y<z$, the remainder when $2025x$ is divided by z is less than the remainder when $2025y$ is divided by z. What is the sum of the elements of S?

设 S 为所有大于 1 的整数 z 所构成的集合, 且对于所有满足 $x<y<z$ 的非负整数对 (x,y), $2025x$ 除以 z 的余数均小于 $2025y$ 除以 z 的余数. 问集合 S 中所有元素的和是多少?

(A) 3041 **(B)** 3542 **(C)** 3750 **(D)** 4044 **(E)** 4319

Problem 24

How many real numbers satisfy the equation $\sin(20\pi x) = \log_{20}(x)$?

问有多少个实数满足方程 $\sin(20\pi x) = \log_{20}(x)$?

(A) 199 **(B)** 200 **(C)** 398 **(D)** 399 **(E)** 400

Problem 25

Three concentric circles have radii 1, 2, 3. An equilateral triangle with side length s has one vertex on each circle. What is s^2?

三个同心圆的半径分别为 1，2，3．一个边长为 s 的正三角形，其三个顶点分别位于三个圆上．问 s^2是多少?

(A) 6 **(B)** $\frac{25}{4}$ **(C)** $\frac{13}{2}$ **(D)** $\frac{27}{4}$ **(E)** 7

2025 AMC 12B Answer Key													
题目	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
答案	B	D	D	A	E	C	C	C	C	A	C	C	D
题目	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
答案	B	D	B	C	D	C	E	C	C	E	D	E	

2025 AMC12B Solution

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